Pergamon

Int. J. Heat Mass Transfer. Vol. 39, No. 4, pp. 753–759, 1996
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0017–9310/96 $15.00+0.00

0017–9310(95)00169–7

Evaluation of the relaxation time of heat and mass exchange in the liquid–vapour bubble flow

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(*Received* 22 *November* 1994 *and in final form* 18 *April* 1995)

Abstract—The paper presents a method of evaluation of a time constant that describes relaxational heat exchange between vapour bubbles and superheated liquid. The Scriven equation of bubble growth solved in a numerical way is used to determine the temperature field in the surroundings of the growing bubble. However, initial and boundary conditions are different from those studied before. The heat transfer relaxation time is determined from variations of the average temperature of the superheated liquid surrounding the bubble. Calculations are performed in both Euler's and Lagrange's descriptions. The relaxation time obtained in the present paper is compared with the relaxation time reevaluated from the *Moby Dick* experiment by means of the homogeneous relaxation model.

1. INTRODUCTION

It is widely recognized that the heat transfer between the liquid and vapour bubbles has a decisive influence on wave propagation in one-component two-phase flow (Bilicki *et al.* [1], Bilicki and Kestin [2], Downar-Zapolski [3]). The simplest model accounting for that fact is the homogeneous relaxation model (HRM). On the grounds of HRM it is possible to explain variations of the pressure distribution in near critical flows [1]. So far the relaxation time θ, which is an important feature of the model, has been determined mainly with the help of dimensional analysis of experimental data. The aim of this paper is to present a theoretical way of assessment of this parameter.

The heat transfer between the superheated liquid and the growing vapour bubble has been widely discussed in the literature (Scriven [4], Hsieh [5], Prosperetti and Plesset [6], Plesset [7]). The initial growth of the bubble nucleus depends strongly on the interfacial mechanical interactions like acceleration, pressure forces and surface tension forces (Madejski and Staniszewski [8]). During this stage thermal phenomena are negligible. Therefore, this stage is called isothermal. As the nucleus radius increases the bubble growth becomes mostly dependent on supply of the heat that is consumed to vaporize the liquid on the bubble surface. During this stage the rate of expansion of the bubble is much lower than during the isothermal stage. The bubble and the liquid surrounding it can be assumed to make up an isobaric system. This stage of the bubble growth is called isobaric. It is worth noticing that duration of the isothermal stage is very short, compared to the isobaric stage.

In the present paper a model set forth by Scriven [4] is used to determine the field of temperature around the vapour bubble.

2. RELAXATION TIME

Any thermodynamic system is described by means of certain thermodynamic parameters such as pressure, temperature, density and others characterizing the state of the system. Among many states of the system, the state of equilibrium is a distinguished one. In this state all thermodynamic parameters have well-defined values, being constant in space and time as long as the system is isolated from its surroundings. Any action of external thermodynamic forces on the system in equilibrium leads to a nonequilibrium state. The system regains its thermodynamic equilibrium spontaneously. However, as a result of interaction with the surroundings its parameters may obtain different values. The transition of the thermodynamic system from nonequilibrium to equilibrium is called *relaxation.* The time during which the transition takes place is called *relaxation time.* It is assumed that the relaxation time is equal to the time in which a thermodynamic parameter changes from its initial value to a value e times lower. This definition comes from the dynamic analysis of phenomena described by a quantity $Y(t)$ decreasing exponentially with time

$$Y(t) = Y(0)\exp\left(-\frac{t}{\theta}\right). \tag{1}$$

The relaxation time θ appears here as a time constant, setting the rate of decay of the quantity $Y(t)$.

The state of the thermodynamic system is usually described by a function of chosen thermodynamic parameters. The relaxation time corresponding to different parameters may be different. The largest value is considered to be the relaxation time of the entire thermodynamic system. In two-phase flow the highest value belongs to the relaxation time related to evaporation or condensation.

NOMENCLATURE

A	nozzle cross-section area [m^2]
a	thermal diffusivity [m^2 s^{-1}]
C	nozzle circumference [m]
c_p	specific heat at constant pressure [J kg^{-1} K^{-1}]
h	enthalpy per unit mass [J kg^{-1}]
h_{lv}	latent heat of evaporation [J kg^{-1}]
p	pressure [Pa]
r	distance from the centre of the vapour bubble [m]
r_k	outer radius, defined by equation (17)
$R(t)$	radius of the vapour bubble [m]
s	dimensionless parameter
t	time [s]
T	temperature [K]
ΔT	superheating of the liquid [K]
V	volume [m^3]
w	velocity [m s^{-1}]
x	dryness fraction (vapour mass fraction)
z	distance from the nozzle inlet [m].

Greek symbols

α	heat exchange coefficient
β	value of parameter s at the bubble boundary
φ	void fraction (vapour volume fraction)
λ	coefficient of thermal conductivity [W m^{-1} K^{-1}]
θ	relaxation time [s]
ρ	density [kg m^{-3}]
σ	surface tension [kg s^{-2}]
ξ	integration variable.

Subscripts

l	liquid
s	value at saturation
v	vapour
∞	value at a large distance from the vapour bubble
0	initial value.

The relaxation time due to the growth of the vapour bubble of initial radius R_0 has been determined from time variations of the average temperature $\bar{T}_l$ of the superheated liquid surrounding the bubble. To represent the bubble surroundings a sphere of outer radius $r_k > R_0$ has been chosen. The average temperature in this domain is a function of time defined as follows:

$$\bar{T}_l(t) = \frac{1}{V_l(t)} 4\pi \int_{R(t)}^{r_k} r^2 T_l(r)\,\mathrm{d}r$$

where

$$V_l(t) = \tfrac{4}{3}\pi(r_k^3 - [R(t)]^3) \tag{2}$$

is the instantaneous volume of the liquid. For a fixed value of r_k, the function $\bar{T}_l(t)$ may be calculated between instants $t_0 = 0$ and t_k, the final instant t_k determined from relation $R(t_k) = r_k$.

The total drop of the average temperature of the liquid is equal to its initial superheating. According to the definition (1), the relaxation time θ_T related to the temperature of the liquid is equal to the time during which the average temperature decreases to

$$\bar{T}_l(\theta_T) = T_s + \Delta T_0\,\mathrm{e}^{-1} \approx T_s + 0.368\Delta T_0. \tag{3}$$

Let the average superheating of the liquid $\overline{\Delta T} = \bar{T}_l - T_s$ be introduced. In Fig. 1, the evaluation of the relaxation time based on time evolution of the average superheating $\overline{\Delta T}$ is presented. It has been assumed that the initial value of void fraction $\varphi_0 = 0.01$, initial superheating of the liquid $\Delta T_0 = 2.0$ K and the initial bubble radius $R_0 = 0.1$ mm.

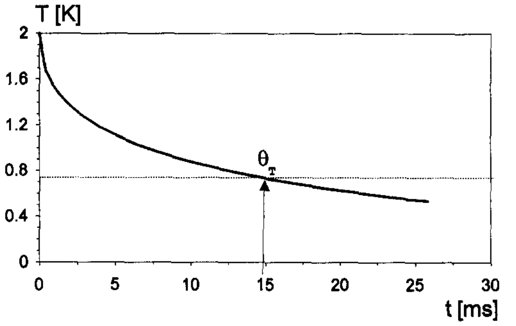

Fig. 1. Determination of the relaxation time θ_T from time evolution of the average liquid superheating $\overline{\Delta T}$ in the surroundings of the vapour bubble.

The relaxation time determined from growth of a single vapour bubble can describe the heat and mass exchange in the entire bubble flow if the flow is homogeneous, that is when the distance between all neighbouring bubbles in a control volume is the same and all bubbles have the same radius.

3. RELAXATION MODEL

In this chapter we concentrate on evaporation, or formation of the vapour phase. The process of evaporation can turn particularly intensive during a rapid pressure drop in the liquid. Then this process is referred to as *flashing*. Correct determination of flashing flow requires taking into account thermodynamic nonequilibrium between the phases. One of the models allowing for nonequilibrium is the homogeneous relaxation model (HRM) [2]. Within the framework of HRM it is assumed that vapour and liquid phases move with equal velocities and that the structure of the flow is a homogeneous mixture of the two phases. Furthermore, it is assumed that the flow is one-dimensional (1D). Let us consider a steady flow through a nozzle of varying cross-section $A = A(z)$ without the influence of mass forces and without heat exchange with the surroundings. The conservation laws for mass, momentum and energy of the homogeneous two-phase mixture can be written in the form

$$\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}z}+\frac{1}{w}\frac{\mathrm{d}w}{\mathrm{d}z}=-\frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}z} \tag{4}$$

$$\rho w\frac{\mathrm{d}w}{\mathrm{d}z}+\frac{\mathrm{d}p}{\mathrm{d}z}=-\tau\frac{C}{A} \tag{5}$$

$$\frac{\mathrm{d}h}{\mathrm{d}z}+w\frac{\mathrm{d}w}{\mathrm{d}z}=0. \tag{6}$$

In HRM, thermodynamic nonequilibrium is described by an additional equation

$$\frac{\mathrm{d}x}{\mathrm{d}z}=-\frac{x-\bar{x}}{\theta_x w}. \tag{7}$$

Equation (7) defines the rate at which the local nonequilibrium dryness fraction x tends toward its local equilibrium value $\bar{x}(p,h)$. In this model the time needed to reach the state of equilibrium is proportional to a parameter θ_x, the relaxation time of the dryness fraction x.

In equations (4)–(7), dependent variables (p,ρ,w,x) are the averaged nonequilibrium parameters of the two-phase mixture. The set of the above equations is supplemented with the state equation

$$h=h(p,\rho,x)=xh_{\mathrm{vs}}(p)+(1-x)h_{\mathrm{l}}(p,T_{\mathrm{l}}(p,\rho,x)) \tag{8}$$

where $h_{\mathrm{vs}}(p)$ denotes the enthalpy of the vapour at saturation and h_{l} denotes the enthalpy of the liquid in a metastable state. A closure equation defines shearing stress at the nozzle wall

$$\tau=\tfrac{1}{2}fw^2\rho \tag{9}$$

where f is an empirical friction factor.

Considering the fact that the decisive influence on the phase formation during evaporation or condensation is attributed to the heat exchange between the two phases, the relaxation equation (7) can be rewritten in another form

$$\frac{\mathrm{d}T_{\mathrm{l}}}{\mathrm{d}z}=-\frac{T_{\mathrm{l}}-T_{\mathrm{vs}}}{\theta_{\mathrm{T}}w}. \tag{10}$$

As the temperature T_{l} and the dryness fraction x are coupled by the state equation, from equations (10) and (7) one can find the following relation between the spatial derivatives of those parameters

$$\frac{\mathrm{d}T_{\mathrm{l}}}{\mathrm{d}z}=\left(\frac{\partial T_{\mathrm{l}}}{\partial h_{\mathrm{l}}}\right)_{\mathrm{P}}\left\{\frac{1}{1-x}\frac{\mathrm{d}h}{\mathrm{d}z}-\frac{h+h_{\mathrm{vs}}}{(1-x)^2}\frac{\mathrm{d}x}{\mathrm{d}z}-\left[\frac{x}{1-x}\frac{\mathrm{d}h_{\mathrm{vs}}}{\mathrm{d}p}+\left(\frac{\partial h_{\mathrm{l}}}{\partial p}\right)_{\mathrm{T_l}}\right]\frac{\mathrm{d}p}{\mathrm{d}z}\right\}. \tag{11}$$

The relaxation time θ_x or θ_{T} is an additional function depending on the thermodynamic state of the system and should be defined by a separate equation. Formulation of this equation comes across certain difficulties. One way to determine the relaxation time is dimensional analysis and construction of a correlation with experimental data [3]. A method to evaluate θ_x has been proposed in [1]. It is based on the fact that knowing the distribution of void fraction $\varphi(z)$ along the nozzle from experimental measurements one can eliminate the dryness fraction from equation (7) with the help of the following relation

$$x=\varphi\frac{\rho_{\mathrm{vs}}}{\rho}. \tag{12}$$

Then, after certain simplifications the relaxation time may be calculated as

$$\begin{aligned}\frac{1}{\theta_x}&=-\frac{w}{x-\bar{x}}\frac{\mathrm{d}x}{\mathrm{d}z}\\&=\frac{w}{\bar{x}-\varphi\dfrac{\rho_{\mathrm{vs}}}{\rho}}\left(\frac{\rho_{\mathrm{vs}}}{\rho}\frac{\mathrm{d}\varphi}{\mathrm{d}z}-\varphi\frac{\rho_{\mathrm{vs}}}{\rho^2}\frac{\mathrm{d}\rho}{\mathrm{d}z}+\frac{\varphi}{\rho}\frac{\mathrm{d}\rho_{\mathrm{vs}}}{\mathrm{d}p}\frac{\mathrm{d}p}{\mathrm{d}z}\right).\end{aligned} \tag{13}$$

In the preceding equation ρ_{vs} denotes the density of the vapour at saturation. Equation (13) together with the conservation equations (4)–(6) and with the state equation (8) constitute a closed set of equations to calculate the function $\theta_x(\varphi(z))$, pressure, density and velocity distributions, and indirectly also the dryness fraction in nonequilibrium two-phase flow in the nozzle. Additionally, the distribution of the relaxation time $\theta_{\mathrm{T}}(\varphi(z))$ of heat exchange can be obtained from equations (10) and (11).

4. FORMULATION OF THE HEAT EXCHANGE PROBLEM

In this chapter an attempt is made to theoretically evaluate the relaxation time θ_{T} related to evaporation in a two-phase medium. We are concerned with the changes of temperature in the surroundings of the vapour bubble growing in the superheated liquid. Let us refer to an equation proposed by Scriven [4] to describe dynamics of a spherical bubble growing in a homogeneous, quiescent liquid of infinite volume. The equation is valid during the isobaric stage when the influence of surface tension and pressure difference between the phases can be neglected. Other effects like vapour compressibility and viscosity, which under all but the most extreme conditions exert a minor influence on the rate of growth of the bubble, are also disregarded. Furthermore, pressure and temperature gradients inside the bubble are neglected and the invariability of physical properties of the liquid and vapour is assumed. As a result of these simplifications the vapour is at the saturation temperature.

Because of the spherical symmetry of the local temperature field T_l a spherical system of coordinates is adopted with the origin in the bubble centre. With respect to the preceding assumptions and following from the equations of conservation of mass, momentum and energy Scriven [4] derived an equation which describes the distribution of temperature $T_l(r, t)$ in the liquid surrounding the bubble

$$\frac{\partial T_l}{\partial t} + \varepsilon \dot{R} \frac{R^2}{r^2} \frac{\partial T_l}{\partial r} = a_l \left(\frac{\partial^2 T_l}{\partial r^2} + \frac{2}{r} \frac{\partial T_l}{\partial r} \right) \tag{14}$$

with a parameter

$$\varepsilon = 1 - \frac{\rho_v}{\rho_l}$$

and the Lagrangean velocity of the bubble surface

$$\dot{R} = \frac{\mathrm{d}R}{\mathrm{d}t}.$$

Equation (14) together with the following boundary conditions

$$T_l(R, t) = T_s \quad T_l(\infty, t) = T_\infty \tag{15}$$

and the initial condition

$$T_l(r, 0) = T_\infty \tag{16}$$

will thereafter be called the Scriven's model of growth of the vapour bubble.

From the practical point of view it is useful to assume that in bubble two-phase flow there exists a finite number of bubbles submerged in the superheated liquid. The distance between neighbouring bubbles depends on their concentration and is usually small. Therefore, growth of an individual bubble due to evaporation of the superheated liquid is determined by heat flux from its close surroundings as heat from more distant regions is consumed by other vapour bubbles present in the two-phase mixture under consideration. Thus, the boundary of the region which has a direct influence on dynamics of a single bubble is a surface on which there is no heat flux (let it be a sphere of radius r_k). That fact is not incorporated in the Scriven's model but may be taken into account by a proper modification of the boundary conditions. Noting that the temperature of interfacial surface is determined by the heat flux q through this surface

$$q = \lambda_l \left(\frac{\partial T_l}{\partial r} \right)_{r=R} = \alpha [T_l(R, t) - T_l(r_k, t)] = \rho_v h_{lv} \dot{R} \tag{17}$$

equation (14) is augmented now by initial and boundary conditions in the form of

$$\left(\frac{\partial T_l}{\partial r} \right)_{r=R} = \frac{\alpha}{\lambda_l} [T_l(R, t) - T_l(r_k, t)], \quad \left(\frac{\partial T_l}{\partial r} \right)_{r=r_k} = 0 \tag{15a}$$

$$T_l(r, 0) = T_s + \Delta T_0 \tag{16a}$$

where α is the coefficient of the heat exchange on the interfacial surface and λ_l is the coefficient of thermal conductivity of the liquid. Scriven's equation (14) together with the conditions (15a) and (16a) will thereafter be referred to as the modified model. The coefficient α is calculated from equation (17) with the help of the following relation for the velocity of the bubble surface

$$\dot{R} = \sqrt{\frac{3}{\pi}} Ja \sqrt{\left(\frac{a_l}{t + t_r} \right)} \tag{18}$$

in which Ja is the Jakob number and t_r is a constant determining the initial growth velocity. Equation (18) is an approximation valid during the isobaric stage [9].

The radius r_k is evaluated from the simplifying assumption that the nuclei of respective bubbles are distributed in the nodes of a 3D grid consisted of cubic cells of side d. Furthermore, the distance between the nuclei is assumed to be constant in time. As a result of heat transfer from the superheated liquid surrounding a nucleus, evaporation takes place on its surface and the nucleus becomes a growing vapour bubble. It follows from the assumed nuclei distribution that volume assigned to a single nucleus is equal to the volume of the elementary grid cell, that is a cube of side d. This observation allows us to determine the radius r_k in equation (15a)

$$d^3 = \tfrac{4}{3} \pi r_k^3 \Rightarrow r_k = d \sqrt[3]{\left(\frac{3}{4\pi} \right)} \approx 0.62d. \tag{19}$$

The radius of the bubble nucleus R_0 at the initial moment $t_0 = 0$ can be calculated from

$$R_0 = \frac{2\sigma}{\rho_v h_{lv}} \frac{T_s}{\Delta T_0} + \delta \tag{20}$$

where δ is a deviation of the nucleus radius from its equilibrium value [8]. Thus the initial void fraction in the flow can be defined as

$$\varphi_0 = \frac{V_{0v}}{V_l + V_{0v}} = \frac{R_0^3}{r_k^3}. \tag{21}$$

The void fraction takes its minimum value when R_0 has its equilibrium value, i.e. $\delta = 0$ in equation (20). It is assumed that the maximum void fraction for which proposed models are valid is equal to $\varphi_{\max} = 0.7$. For higher values neighbouring bubbles would merge and the flow pattern would change.

5. SOLUTION OF HEAT EXCHANGE PROBLEM

The problem of heat exchange between the vapour bubble and the superheated liquid described by the Scriven's model [equations (14)–(16)] can be solved by means of the similarity transformation. To be precise, it is assumed that the solution of equation (14) can be written in the form

$$T_l(r,t) = T_l(s) \tag{22}$$

where s is a nondimensional time-independent parameter defined by the equation

$$s = \frac{r}{2\sqrt{(a_l t)}}. \tag{23}$$

As a result of this transformation the following ordinary differential equation is obtained

$$\frac{d^2 T_l}{ds^2} = 2\left(-s - \frac{1}{s} + \frac{\varepsilon\beta^3}{s^2}\right)\frac{dT_l}{ds} \tag{24}$$

where β is a value of the parameter s on the bubble surface, or $\beta = s(R,t)$. Equation (24) integrated twice yields

$$T_l(s) = T_\infty - 2\frac{\rho_v h_{lv}}{\rho_l c_{pl}}\beta^3 \exp(\beta^2 + 2\varepsilon\beta^2)\int_s^\infty \frac{1}{\xi^2} \times \exp\left(-\xi^2 - 2\frac{\varepsilon\beta^3}{\xi}\right)d\xi. \tag{25}$$

It follows from the first boundary condition (15) that $T_l(\beta) = T_s$. Thus, from equation (25) an integral equation can be obtained

$$\Delta T_\infty = 2\frac{\rho_v h_{lv}}{\rho_l c_{pl}}\beta^3 \exp(\beta^2 + 2\varepsilon\beta^2)\int_\beta^\infty \frac{1}{\xi^2} \times \exp\left(-\frac{1}{\xi^2} - 2\frac{\varepsilon\beta^3}{\xi}\right)d\xi \tag{26}$$

from which β should be calculated. $\Delta T_\infty = T_\infty - T_s$ denotes here superheating of the liquid at a great distance from the vapour bubble. The equation above should be solved numerically.

Since β is determined the nondimensional superheating function $\Delta T(s)$ can be found from equation (25). Making use of equation (23) the function $\Delta T(s)$ can be transformed into the superheating temperature distribution $\Delta T(r)$ for any chosen instant t. In Fig. 2, the function $\Delta T(s)$ is shown together with $\Delta T(r)$ for chosen instants $t_1 < t_2 < t_3 < t_4 < t_5$.

The similarity transformation can not be applied to solve the equations of the modified model [equations (14), (15a), (16a)]. The temperature field described by thismodel should be found in a numerical way. An implicit finite difference scheme of Briley–McDonald has been used to perform the calculations in the Lagrangean coordinates.

6. RESULTS OF NUMERICAL CALCULATIONS AND COMPARISON WITH EXPERIMENT

The relaxation time θ_T has been evaluated for a growing vapour bubble in water under atmospheric pressure ($p_\infty = 1.01$ bar, $T_s = 373.15$ K). The following data have been used for calculations: $a_l = 16.9 \times 10^{-8}$ m^2 s^{-1}, $c_{pl} = 4216$ J kg^{-1} K^{-1}, $h_{lv} = 22.56 \times 10^5$ J kg^{-1}, $\rho_l = 958.4$ kg m^{-3}, $\rho_v = 0.598$ kg m^{-3}, $\lambda = 0.6825$ W m^{-1} K^{-1}, $\sigma = 0.0587$ kg s^{-2}.

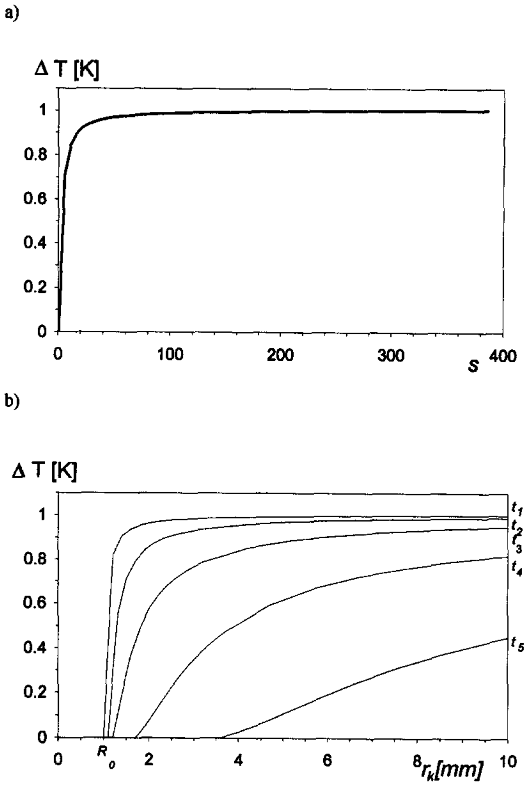

Fig. 2. Distribution of the liquid superheating $\Delta T = T_l - T_s$ around the vapour bubble: (a) as a function of the nondimensional parameter s; (b) as a function of the distance from the bubble centre, for chosen instants $t_1 < t_2 < t_3 < t_4 < t_5$. R_0 indicates the bubble radius at the initial moment $t_0 = 0$ and r_k is a final radius of the bubble.

The dependence of the initial void fraction φ_0 on the relaxation time θ_T is calculated for two values of the liquid superheating: $\Delta T_{01} = 1$ K and $\Delta T_{02} = 2$ K. The initial equilibrium bubble radius is then equal to $R_{01} = 0.0325$ mm and $R_{02} = 0.0162$ mm respectively. The results of calculations performed for $d = 4$ mm are presented in Fig. 3. The obtained values of the relaxation time θ_T depend on the initial superheating of the liquid ΔT_0, namely θ_T decreases with the increase in the superheating. Moreover, a rapid decrease in θ_T is observed when the initial void fraction increases. Both discussed models agree well for the small initial superheating of the liquid [Fig. 3(a)] but the results differ somehow with the rise in the initial superheating [Fig. 3(b)].

The influence of turbulence on the relaxation time has also been investigated. It is well recognized that turbulence intensifies heat transfer. In the models under study, turbulence in the liquid surrounding vapour bubble can be simulated by the increase in the thermal diffusivity a_l to account for an increase in the heat flux through the interfacial surface. This, in turn, should increase evaporation and accelerate the bubble

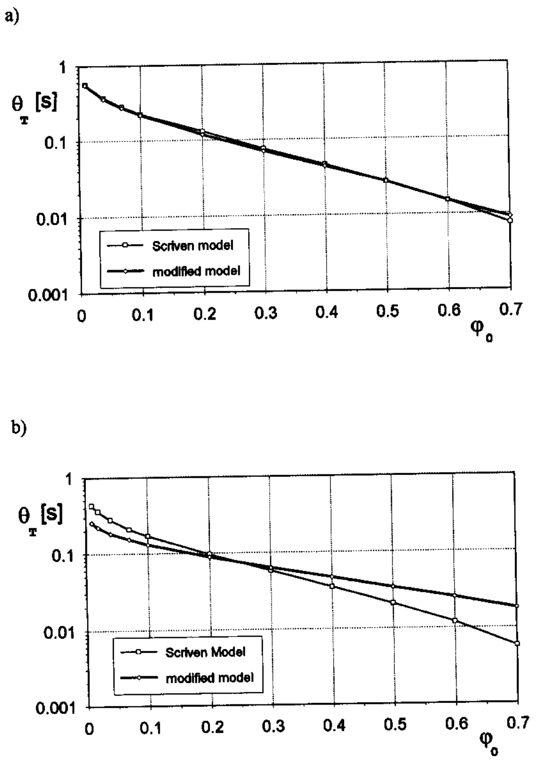

Fig. 3. The relaxation time θ as a function of the initial void fraction φ_0 according to the presented models for different values of the superheating of the liquid: (a) 1K, (b) 2K.

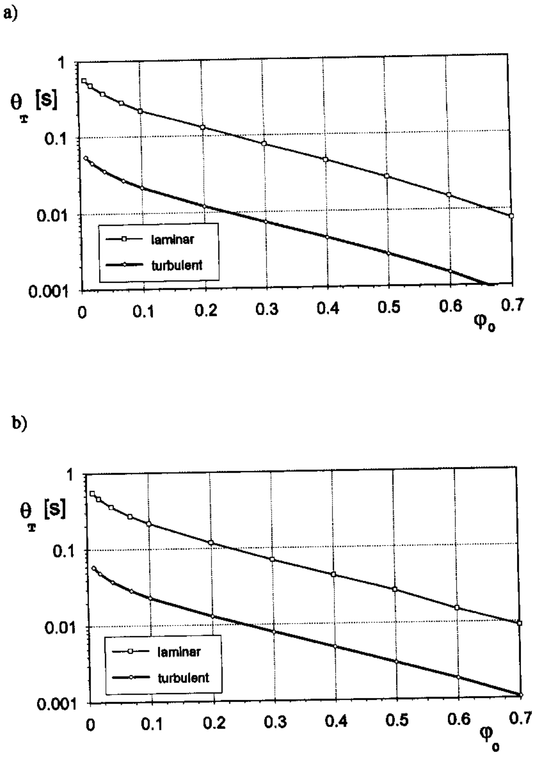

Fig. 4. Influence of the thermal liquid diffusivity a_l on the relaxation time: (a) in the Scriven's model and (b) in the modified model. The thermal diffusivity in laminar flow is $a_l = 16.9 \times 10^{-8}\ \mathrm{m^2\,s^{-1}}$, in turbulent flow $a_l = 17 \times 10^{-7}\ \mathrm{m^2\,s^{-1}}$.

growth. Therefore, lower values of the relaxation time are expected. The expectation is confirmed by calculations whose results are presented in Fig. 4. The functions $\theta(\varphi_0)$ shown there for two different values of the thermal diffusivity refer to the superheating $\Delta T = 1.0$ K and to the distance between the bubbles $d = 4$ mm. For a qualitative study of turbulent flow it has been assumed somehow arbitrarily that the thermal diffusivity of the liquid a_l is 10 times higher than in laminar flow.

Predictions of both discussed models have also been compared with values of the relaxation time calculated from the *Moby Dick* [10] measurements of two-phase flow in a divergent nozzle. A method presented in [1] has been used to evaluate the "experimental" relaxation time θ_T. The calculations were based on equations of HRM with the evolution equation (10) and were performed in a way similar to that described in Section 3. A comparison is presented in Fig. 5. Values of the relaxation time obtained from the *Moby Dick* experiment (run 400) are plotted as black squares ■. Each value of the void fraction corresponds to a different location in the nozzle. Therefore, each value of the "experimental" relaxation time refers to a different thermodynamic state.

The comparative calculations were performed under conditions similar to those in the nozzle throat in run 400 of the *Moby Dick* experiment, namely $p = 1.5$ bar, $T_s = 384.528$ K, $\Delta T = 4.0$ K, $c_l = 4225\ \mathrm{J\,kg^{-1}}$, $h_{lv} = 22.26 \times 10^5\ \mathrm{J\,kg^{-1}}$, $\rho_l = 949.94\ \mathrm{kg\,m^{-3}}$, $\rho_v = 0.8624\ \mathrm{kg\,m^{-3}}$, $\beta = 0.9709244$. As neither the thermal diffusivity a_l nor the distance between the bubble nuclei d were determined in the experiment, their values were selected arbitrarily. For a fixed value of a_l, the distance d was chosen to obtain the best agreement with the experimental data. For a laminar value of $a_l = 17.0 \times 10^{-8}\ \mathrm{m^2\,s^{-1}}$ we felt that the most suitable distance between the bubble nuclei was $d = 7$ mm. The results of calculations are presented in Fig.

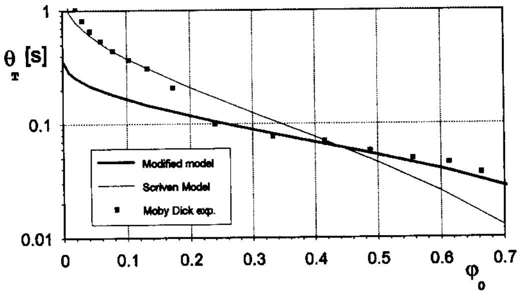

Fig. 5. Comparison of the values of the relaxation time calculated on the grounds of the discussed models and the *Moby Dick* experiment (run 400).

5. Similar curves can be obtained for turbulent flow provided that a new value of d is appropriately selected.

7. CONCLUSIONS

The relaxation time evaluated on the grounds of the Scriven's theory of growth of the vapour bubble refers to evaporation on the interfacial surface of a single bubble. The relaxation time θ_T decreases with the increase in the initial void fraction. Lower values of θ_T are also obtained when the superheating of the liquid increases and for turbulent flow as modelled by a higher thermal diffusivity of the liquid a_l.

Modifications of the boundary conditions, presented in the paper, in the Scriven's model have led to a noticeable difference in evaluation of the relaxation time if the liquid superheating exceeds $\Delta T = 1\text{K}$.

A good agreement with the experimental data [10] was observed with respect to the relaxation time θ_T evaluated with the aid of the Scriven's model if the initial void fraction $\varphi_0 < 0.4$. For higher values of the void fraction the predictions of the modified model are more accurate. On the other hand, the modified model predicts too low values of the relaxation time for small values of φ_0. It is believed that a reason for this lies in the simplified relation (18) for the velocity of the bubble surface. This relation is an approximate solution of the problem of heat exchange between the vapour bubble and the superheated liquid and should be improved for small bubbles.

Acknowledgements—The authors wish to acknowledge the financial support received from the State Committee for Scientific Research under Grant PB 0376/P4/94/06.

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